Improved Formulae for the Inductance of Straight Wires

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Abstract

The best analytical formulae for the self-inductance of rectangular coils of circular cross section available in the literature were derived from formulae for the partial inductance of straight wires, which, in turn, are based on the well-known formula for the mutual inductance of parallel current filaments, and on the exact value of the geometric mean distance (GMD) for integrating the mutual inductance formula over the cross section of the wire. But in this way, only one term of the mutual inductance formula is integrated, whereas it contains also other terms. In the formulae found in the literature, these other terms are either completely neglected, or their integral is only coarsely approximated. We prove that these other terms can be accurately integrated by using the arithmetic mean distance (AMD) and the arithmetic mean square distance (AMSD) of the wire cross section. We present general formulae for the partial and mutual inductance of straight wires of any cross section and for any frequency based on the use of the GMD, AMD, and AMSD.

Since partial inductance of single wires cannot be measured, the errors of the analytical approximations are computed with the help of exact computations of the six-dimensional integral defining induction. These are obtained by means of a coordinate transformation that reduces the six-dimensional integral to a three-dimensional one, which is then solved numerically. We give examples of an application of our analytical formulae to the calculation of the inductance of short-circuited two-wire lines. The new formulae show a substantial improvement in accuracy for short wires.

1. Introduction

The self-inductance of a straight wire may only be defined as so-called partial inductance [1, 2]. Per se, partial inductance of a single wire cannot be measured. Only loop inductance can be measured. Therefore, partial inductance can only be calculated or determined indirectly from measurements of loop inductance via calculations.

Partial self-inductance of a conductor is defined as the double volume integral of the scalar product of the current density vectors $\vec{J_1}(\vec{r_1})$ and $\vec{J_2}(\vec{r_2})$ at the points $\vec{r_1}$ and $\vec{r_2}$ divided by the distance r_{12} between these points, carried out over the whole volume of the conductor,

$$L = \frac{\mu_0}{4\pi}\frac{1}{I^2}\iint \vec{J_1}\cdot\vec{J_2}\frac{d\tau_1 d\tau_2}{r_{12}}\,, \qquad (1)$$

where $d\tau_1$ and $d\tau_2$ are the volume elements around the integration points $\vec{r_1}$ and $\vec{r_2}$, respectively, and where for simplicity we assume non-magnetic conductor material, so that μ_0 is the magnetic permeability of the vacuum, $\mu_0 = 4\pi \cdot 10^{-7} Vs/(Am)$, and I is the total current flowing in the conductor (see equation (17a), p. 95 in [3]).

In this paper, we present two methods that allow deriving analytic formulae for the partial inductance of straight wires *of any cross section* and *for any frequency*. We apply these methods to derive formulae for the cases of circular cross section in the low-frequency limit (where the current distribution is homogeneous) and in the high-frequency limit (where the current is concentrated on the surface of the wire). A couple of formulae for the partial inductance of wires of circular cross section in the low-frequency limit can be found in the literature. We want to compare the accuracies of the various formulae. Since partial inductance cannot be measured, we cannot rely on measurements to assess the analytical results.

But fortunately, for wires of circular cross section in the low-frequency limit it is possible to calculate the partial inductance *exactly* as the six-dimensional integral of the general inductance definition (1). Instead of taking measurements, we take recourse to such calculations. We present a transformation of coordinates which allows reducing the six-dimensional integral (1) to a three-dimensional one. The three-dimensional integral can be computed by means of the function `integral3` which forms part of the MATLAB® programming language.

The analytical formulae which can be found in the literature are all based on the fact that the integration along the wire (i.e. in the direction of current flow) in equation (1) can be carried out in closed form. The integration is done along the longitudinal coordinates z_1 and z_2 of the points $\vec{r_1}$ and $\vec{r_2}$ which can independently assume any position along the wire. The result of this two-dimensional integration is the well-known formula for the mutual inductance M of two straight parallel filaments of equal length l separated by a distance δ [1 - 5]: