

# Multiband Slot- and Stub-Based Dual Composite Right-/Left-Handed Transmission Line

Elena Abdo-Sánchez\*, Teresa M. Martín-Guerrero and Carlos Camacho-Peñalosa

Departamento de Ingeniería de Comunicaciones, E. T. S. Ingeniería de Telecomunicación,  
Universidad de Málaga, Campus de Teatinos, 29071 Málaga, Spain.

\*E-mail: elenaabdo@ic.uma.es

## Abstract

A dual Composite Right-/Left-Handed Transmission Line implementation that presents multiband behaviour is proposed in this contribution. The artificial transmission line is implemented by loading a host microstrip line with alternate rectangular stubs and slots. The required series and shunt immittances are respectively provided by the slot and the stub. Because of the distributed nature of these immittances, the resulting phase response presents theoretically-infinite Right-Handed and Left-Handed alternate bands, thus being appropriate for multiband applications. The design methodology is described with the help of a proposed transmission line-based equivalent circuit and highlights the simplicity for balanced condition. Full wave simulated results of the dispersion characteristics and frequency response of a unit-cell and a three-cell structure are presented.

## 1. Introduction

Although metamaterials have been studied theoretically since the appearance of Veselago's paper [1] and first implemented at microwave frequencies by Pendry [2] and Smith [3], their lossy and resonant behaviour limited their practical application in microwave components until 2002. Then, three different research groups developed the Transmission Line (TL) approach of metamaterials [4, 5, 6]. This new approach allowed countless applications in microwave engineering to arise. Most of them are based on the so-called Composite Right-/Left-Handed Transmission Line (CRLH TL), which behaves as an LH (Left-Handed) medium at low frequencies and as RH (Right-Handed) at higher frequencies under certain assumptions. After the introduction of the CRLH TL, artificial TLs with different topologies were proposed, such as the 'dual' [7] or the 'extended' [8]. Even a derivation of artificial TLs of arbitrary order was presented in [9]. In this way, a specific dispersion diagram can be synthesized by simply cascading the necessary unit-cells.

Several implementations of artificial TLs with high order in planar technology can be found in the literature. Recently, a planar dual CRLH TL based on the use of Defected Ground Structure (DGS) with an inter-digital gap was demonstrated [10]. An alternative to this structure was presented by the authors in [11], which makes use of al-

ternate dumbbell-shaped DGSs and stubs for building a tri-band artificial TL and has the advantage of a simple design for the balanced case. However, its important stop band and its lack of matching over the rest of the studied bandwidth have limited its possible applications. In this contribution we present a variant of the aforementioned structure, which uses alternate rectangular stubs and slots for providing the required shunt and series immittances, respectively. The proposed unit-cell presents an improved frequency behaviour, with reduced stop bands. Moreover, the analysis of the structure and design methodology are provided here in terms of the distributed immittances, instead of lumped elements, thus extending its validity and being more accurate.

This contribution is organized as follows. Section 2 describes the geometry of the proposed unit-cell and its equivalent circuit using TLs. Section 3 deals with the design methodology for achieving a balanced CRLH TL. In Section 4, the results for a design example of unit-cell and for the corresponding three-cell CRLH TL are shown. Finally, Section 5 summarizes the main conclusions.

## 2. Unit-Cell Geometry And Equivalent Circuit

The proposed unit-cell is shown in Fig. 1. It consists of a host microstrip TL loaded with a rectangular stub and a rectangular slot etched on its ground plane. In the figure,  $w_H$  stands for the width of the host microstrip line;  $w_M$  and  $l_M$ , for the stub width and length, respectively, and  $w_S$  and  $l_S$  represent the slot width and length, respectively. In addition,  $p$  stands for the unit-cell length.

A slot etched on the ground plane of a microstrip line behaves as a series impedance. This impedance can be approximated to that presented by the corresponding slotline terminated in short circuit. Likewise, a stub behaves as a shunt admittance which is equivalent to that presented by the corresponding microstrip line terminated by an open circuit. Therefore, the slot and stub equivalent impedances can be written as follows:

$$Z_{slot} = -\frac{j}{2} Z_S \cot \frac{\theta_S}{2} \quad (1a)$$

$$Z_{stub} = \frac{j}{2} Z_M \tan \frac{\theta_M}{2} \quad (1b)$$

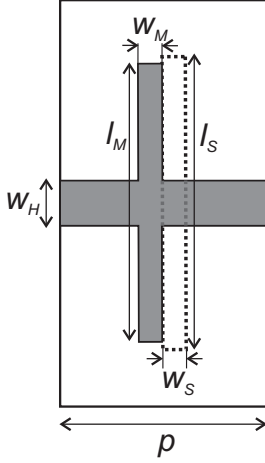


Figure 1: Unit-cell geometry.

where  $Z_S$  and  $Z_M$  are the characteristic impedances and  $\theta_S/2$  and  $\theta_M/2$ , the electrical lengths of the corresponding slotline and microstrip TLs. For a given substrate, these parameters depend on the microstrip ( $w_M$ ) and slotline ( $w_S$ ) widths [12]. The resultant impedances in (1a) and (1b) are the result of the parallel connection of two slotlines terminated in short circuit and two microstrip lines terminated in open circuit. The physical lengths of these transmission lines would be  $1/2(l_M - w_H)$  for the stub and  $1/2l_S$  for the slot, since the stub is measured from the edge of the host microstrip line and the slot from the middle. However, the short and open circuits effects lengthen the slotline and microstrip line, respectively [12]. Therefore, the equivalent lengths will be slightly greater than the physical ones.

A straightforward equivalent circuit for the proposed unit-cell is shown in Fig. 2, where  $Z_H$  and  $\theta_H$  stand for the characteristic impedance and electrical length of the host microstrip line, respectively. In this equivalent circuit, the mutual coupling effects between the stub and the slot and the host microstrip interconnection section between them have been neglected.

### 3. Design

By ignoring the host microstrip sections, the series impedance of the unit-cell corresponds to (1a) and the shunt admittance, to the inverse of (1b). In order to achieve a ‘balanced condition’ (i.e., no stop bands around the zeros of the phase response [4]) they must present exactly the same critical frequencies (poles and zeros). In this way, two conditions must be fulfilled for the balanced case:

$$\theta_S = \theta_M \quad (2a)$$

$$Z_H = \frac{1}{2} \sqrt{Z_M Z_S}. \quad (2b)$$

It can be assumed that the widths of the stub and slot are the practically only geometric variables that determine the characteristic parameters of the equivalent microstrip

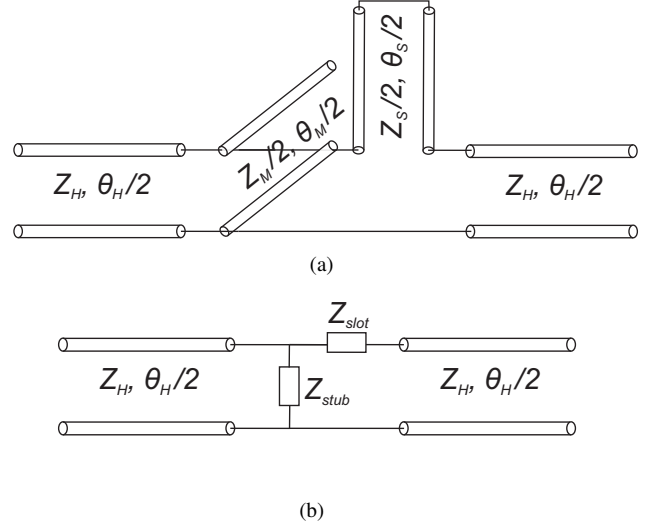


Figure 2: Equivalent circuit of the proposed unit-cell of Fig. 1.(b) is a simplified version of (a).  $Z_{slot}$  and  $Z_{stub}$  are defined in (1a) and (1b), and  $Z_S$ ,  $Z_M$ ,  $\theta_S$  and  $\theta_M$ , in the text below them.

and slotline, respectively. This means that, once the widths of the elements are chosen, the parameters  $Z_M$ ,  $\varepsilon_M$ ,  $Z_S$  and  $\varepsilon_S$  are already determined, where  $\varepsilon_M$  and  $\varepsilon_S$  are the effective permittivities corresponding to the stub and the slot, respectively. Then, the lengths of the elements can be adjusted to make them present the same poles and zeros. Therefore, the design process can be reduced to a few steps:

- Choose a pair of element widths  $w_M$  and  $w_S$  to obtain the desired  $Z_H$  according to (2b) or, alternatively, adjust the width of the host transmission line  $w_H$  to satisfy (2b).
- Choose the slot length to make it resonate at the desired frequency.
- Adjust the stub length, taking into account the different effective permittivities of the slot and stub, to satisfy (2a).

A unit-cell example on ARLON 1000 substrate with  $\varepsilon_r = 10$  and  $h=50$  mil has been designed. We have chosen this substrate in order to reduce as much as possible the radiation losses of the slot, which can be achieved with thick substrates with high permittivity. For this substrate it is possible to obtain the required characteristic impedance of the host TL for the balance condition as a function of the stub and slot widths, in accordance with (2b). This is shown in Fig. 3 for the ARLON 1000 substrate at 4 GHz. Although a  $50\Omega$ -characteristic impedance can be achieved using reasonable widths on this substrate, we have preferred to choose equal slot and stub widths, since for  $50\Omega$  the slot width has to be higher, thus introducing more radiation losses. Therefore, we have chosen  $w_S = w_M = 0.3$  mm, for which the required characteristic impedance of the host

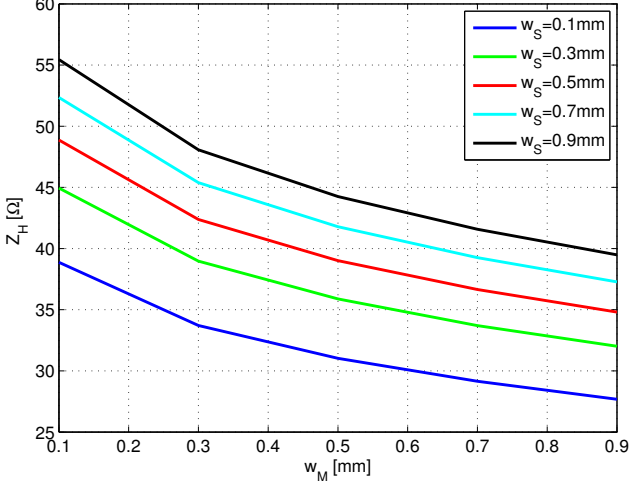


Figure 3: Required characteristic impedance of the host transmission line at 4 GHz as a function of the slot and stub widths, according to (2b).

TL is  $39 \Omega$ , approximately. Therefore, we have adjusted the host TL width to present  $39 \Omega$ , that is  $w_H = 1.95$  mm. Next, the slot length has been set to 30 mm in order to get the first resonance below 3 GHz ( $f_{r1}=2.4$  GHz) and to obtain several LH-RH transitions in a reasonable bandwidth. Then, the stub length has been adjusted to fulfill (2a). To do that, we use the effective permittivities of the corresponding slotline and microstrip TL to theoretically make an approximation of the required stub physical length. However, an optimization step is needed because of the aforementioned short and end effects, which are not considered in the proposed equivalent circuit in Fig. 2. The resultant stub length is 27.25 mm. The geometric parameters of the design are summarized in Table 1.

	Host microstrip	Stub	Slot
Width	1.95 mm	0.30 mm	0.30 mm
Length	–	27.25 mm	30.00 mm
$Z_H, Z_M, Z_S$	$39.0 \Omega$	$85.4 \Omega$	$71.4 \Omega$

Table 1: Summary of the design geometric parameters and characteristic impedances @ 4 GHz.

#### 4. Results

The design example, described in Section 3 has been analysed by means of full-wave electromagnetic simulation. Fig. 4 shows the S-Parameters of the unit-cell. It can be observed that good matching (better than 10 dB) is achieved over the whole band, with the exception of some small stop bands located at the resonance frequencies of the slot/stub. Because of the different frequency dispersion of the equivalent circuit parameters of the slot and stub (characteristic impedance and effective permittivity of the corresponding slotline and microstrip TL), the possibility of achieving

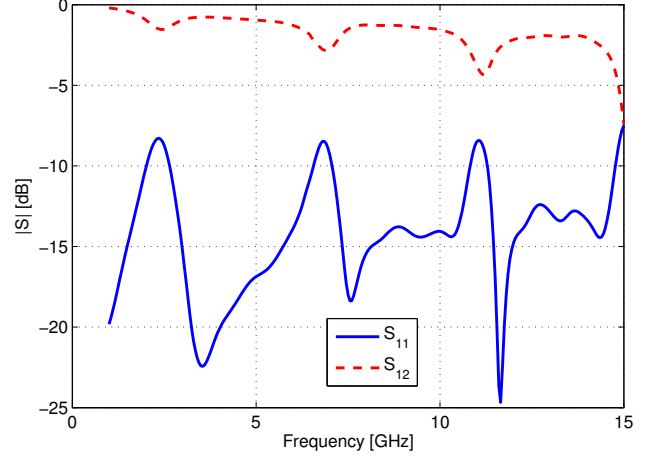


Figure 4: S-Parameters of the unit-cell design example.

good matching in the whole band is limited. This effect is more pronounced at the resonance frequencies, where the equivalent slot and stub impedances in (1a) and (1b) present a zero and a pole, respectively.

Fig. 5 shows the phase factor (i.e., the phase of  $S_{21}$  provided the unit-cell is loaded by its image impedance) of a unit-cell for different cell lengths  $p$ . The curve for  $p = 0$  represents the phase factor of the intrinsic unit-cell, that is, the phase that the slot-strip introduces without including the phase that comes from the sections of the microstrip feeding TL that separate adjacent cells. A multiband behaviour can be seen, with several RH-LH and LH-RH transitions, with good balance achievement, since practically no stop bands appear when the phase factor takes the null value. It is worth mentioning that the multiband behaviour refers to the appearance of pass-bands centred at arbitrary frequencies (not necessarily multiples of a certain frequency). This is thanks to the intrinsic unit-cell that modifies the linear phase response of the host microstrip line by introducing non-linear frequency behaviour. Moreover, it can be observed, that as  $p$  increases, the phase factor curve is shifted to the left, thus introducing more phase at the same frequency and losing its non-linearity. This is the effect of the feeding microstrip sections, since for higher lengths the microstrip sections are longer and they contribute to the total phase response to a greater extent. Therefore, the cell length should not be so high that the linear phase response of the host microstrip line disguises the non-linear behaviour of the intrinsic cell.

In order to study the potential of the unit-cell to build a CRLH TL, we have analyzed a three-cell structure. In Fig. 6, the resultant  $S_{11}$  for different cell lengths is depicted. It can be observed that the reflection coefficient strongly depends on the cell length (on the distances between adjacent slot/stub elements), giving rise to a matching worsening when the cells get closer, which can be attributed to coupling effects.

Keeping a compromise between the linearity effect and the worsening of the matching, we have selected  $p =$

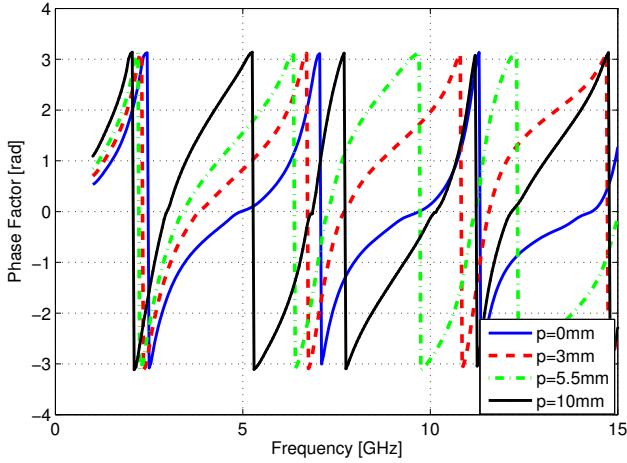


Figure 5: Phase factor for different cell lengths  $p$ .

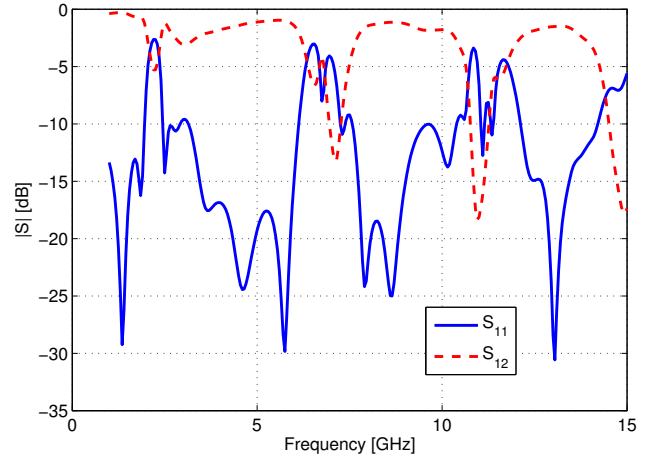


Figure 7: S-Parameters of the three-cell structure with  $p=5.5$ mm.

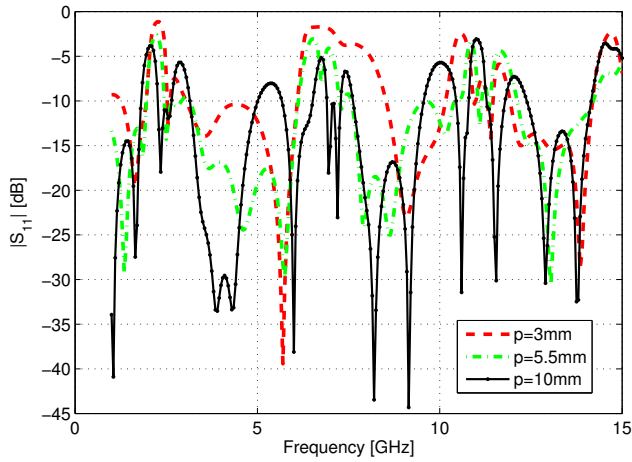


Figure 6:  $|S_{11}|$  of the three-cell structure with different cell lengths.

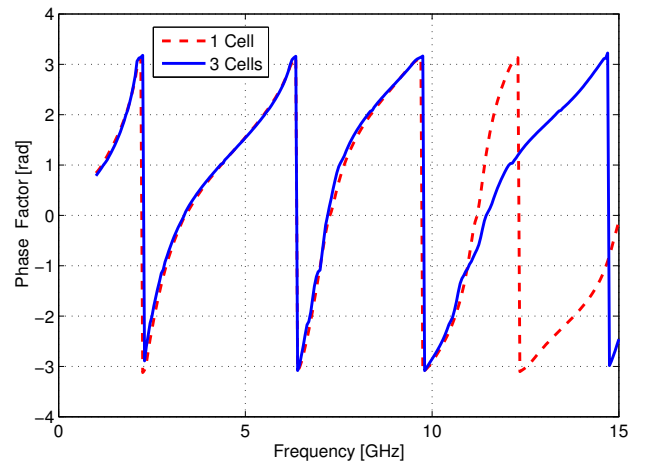


Figure 8: Phase factor of the unit-cell compared to the phase response of the three-cell structure (divided by three), for  $p = 5.5$  mm.

5.5 mm. For this case, Fig. 7 shows the S-Parameters of the resultant three-cell structure. Although, as expected, the behaviour has worsened with respect to that of the unit-cell, three broad pass-bands are obtained, which, according to the corresponding curve (the green one) in Fig. 5, contain both LH and RH behaviour. For example, good matching has been achieved for  $f = 3.4$  GHz, at which a transition from LH to RH occurs, with  $|S_{11}|=-14$  dB and  $|S_{12}|=-2.4$  dB or for  $f = 9.8$  GHz, a transition from RH to LH, with  $|S_{11}|=-10.5$  dB and  $|S_{12}|=-1.8$  dB.

Finally, in order to check whether the phase factor of a single cell can predict the behaviour of an artificial TL made up of several cells, Fig. 8 shows a comparison between the phase factors of the single cell and the three-cell structure. For the sake of comparison, the phase response of the three-cell structure has been divided by the number of cells. Very good agreement is obtained up to 11 GHz. From this frequency, the two curves take different behaviours, which indicates that the TL approach is not valid anymore, since the cell length is in the same order of magnitude as the wave-

length.

## 5. Conclusions

A CRLH TL implementation has been proposed, which makes use of alternate rectangular stubs and slots to provide the required immittances. Since these immittances are distributed, multiband behaviour has been achieved. An equivalent circuit using TLs has been proposed to model the structure behaviour over a broad bandwidth. With the help of this equivalent circuit, the design methodology has been stated. The design simplicity to fulfill the balanced condition has been highlighted.

In order to check the design methodology, simulated results of a unit-cell and a three-cell structure have been analysed. The resultant phase response have confirmed the design methodology, since balanced multiband behaviour has been achieved. Moreover, the structure presents good matching over some broad frequency bands making it quite

appropriate for flexible multiband applications. Since the unit-cell is based on the use of slots, this structure can be a good choice for antenna applications, such as leaky-wave antennas.

### Acknowledgement

This work was supported by the Spanish Ministerio de Ciencia e Innovación (Programa Consolider-Ingenio 2010) under Grant CSD2008-00066, EMET, and by the Junta de Andalucía (Spain) under Grant P10-TIC-6883.

### References

- [1] V. Veselago, "The electrodynamics of substances with simultaneously negative values of  $\epsilon$  and  $\mu$ ," *Soviet Physics Uspekhi*, vol. 10, no. 4, pp. 509–514, Jan.-Feb. 1968.
- [2] J. B. Pendry, A. J. Holden, D. J. Robbins and W. J. Stewart, "Magnetism from conductors and enhanced nonlinear phenomena," *IEEE Transaction on Microwave Theory and Techniques*, vol. 47, no. 11, pp. 2075–2084, Nov. 1999.
- [3] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser and S. Schultz, "Composite Medium with Simultaneously Negative Permeability and Permittivity," *Physical Review Letters*, vol. 84, no. 18., pp. 4184–4187, May 2000.
- [4] C. Caloz and T. Itoh, *Electromagnetics Metamaterials: Transmission Line Theory and Microwave Applications*. Hoboken, NJ: Wiley and IEEE Press, 2006.
- [5] G. V. Eleftheriades, A. K. Iyer, and P. C. Kremer, "Planar negative refractive index media using periodically L-C loaded transmission lines," in *IEEE Transactions on Microwave Theory and Techniques*, vol. 50, no. 12, pp. 2702–2712, Dec. 2002.
- [6] A. A. Oliner, "A planar negative refractive index without resonant elements," in *Proceedings in IEEE-MTTS International Microwave Symposium, Philadelphia, PA*, Jun. 2003, pp. 191194.
- [7] C. Caloz, "Dual Composite Right/Left-Handed (D-CRLH) Transmission Line Metamaterial," *Microwave and Wireless Components Letters, IEEE*, vol. 16, no. 11, pp. 585–587, Nov. 2006.
- [8] A. Rennings, S. Otto, J. Mosig, C. Caloz, and I. Wolff, "Extended composite right/left-handed (E-CRLH) metamaterial and its application as quadband quarter-wavelength transmission line," in *Microwave Conference, 2006. APMC 2006. Asia-Pacific*, Dec. 2006, pp. 1405–1408.
- [9] C. Camacho-Peñalosa, T. M. Martín-Guerrero, J. E. Page, and J. Esteban, "Derivation and General Properties of Artificial Lossless Balanced Composite Right/Left- Handed Transmission Lines of Arbitrary Order," *Progress in Electromagnetics Research B*, vol. 13, pp. 151–169, 2009.
- [10] Y. H. Ryu, J. H. Park, and J. H. Lee, J. Y. Kim, H. S. Tae, "DGS Dual Composite Right/Left Handed Transmission Line," *IEEE Microwave and Wireless Component Letters*, vol. 18, no. 7, pp. 434, July 2008.
- [11] E. Abdo-Sánchez, J. A. Fernández-Segovia, T. M. Martín-Guerrero and C. Camacho-Peñalosa, "Tri-Band Dual CRLH Transmission Line Using Dumbbell-Shaped DGS and Stub", *4th Young Scientist Meeting on Metamaterials*, Valencia, Feb. 2011.
- [12] K. C. Gupta, R. Garg, I. Bahl and P. Bhartia, *Microwave Striplines and Slotlines, 2nd Edition*, Artech House, Norwood, 1996.